An Efficient Method for Study of General Bi-Anisotropic Waveguides

Yansheng Xu, *Member, IEEE*, and Renato G. Bosisio, *Senior Member, IEEE*

***Abstract*—In this paper, coupled-mode equations for general bi-anisotropic waveguides with perfect electrically conducting (PEC) walls are studied and a set of rigorous expressions is obtained. The features of these equations are discussed and applications to circular/rectangular Faraday chiral and chiral waveguides are demonstrated. Comparisons between Faraday chiral, chiral, and ferrite-filled waveguides are also given.**

I. Introduction

A NEW CLASS of waveguides, known as chirowaveguides exhibit novel and unique properties, and they have been studied intensively in recent years. In spite of numerous papers published on this subject [1]–[8], [14], [15], only very few cases have been reported in detail. Moreover, the newly suggested Faraday chiral [16] and general bi-anisotropic waveguides are more complicated to calculate in spite of the fact that they have many interesting properties and potential applications. In reference [24], calculation of circular open chirowaveguides is made by using coupling mode expansion method. Their results are valid only for small chirality cases since the formulas used in [24] are derived with perturbation approximation of small chiral admittance. In this paper, we give a rigorous study of the coupled-mode equations of general bi-anisotropic waveguides with PEC walls including chiral and Faraday chiral waveguides in special cases. It is shown by sample calculations that this rigorous method is very effective to calculate results of both circular and rectangular chiral and Faraday chiral waveguides with various values of chiral admittance and off-diagonal component of permeability tensor. A comparison is also made between chirowaveguides and ferrite-filled waveguides.

II. Theory

The electromagnetic field components of a metal waveguide, filled with bi-anisotropic materials may be expressed as follows [17]:

$$E_u = \sum_n \left(V_{(n)} \frac{\partial \Pi_{(n)}}{h_1 \partial u} + V_{[n]} \frac{\partial \Pi_{[n]}}{h_2 \partial v} \right)$$

$$E_v = \sum_n \left(V_{(n)} \frac{\partial \Pi_{(n)}}{h_2 \partial v} - V_{[n]} \frac{\partial \Pi_{[n]}}{h_1 \partial u} \right)$$

$$H_v = \sum_n \left(I_{(n)} \frac{\partial \Pi_{(n)}}{h_1 \partial u} + I_{[n]} \frac{\partial \Pi_{[n]}}{h_2 \partial v} \right)$$

$$H_u = \sum_n \left(-I_{(n)} \frac{\partial \Pi_{(n)}}{h_2 \partial v} + I_{[n]} \frac{\partial \Pi_{[n]}}{h_1 \partial u} \right)$$

$$E_z = \sum_n \left(\chi_{(n)} V_{z,(n)} \Pi_{(n)} \right)$$

$$H_z = \sum_n \left(\chi_{[n]} I_{z,[n]} \Pi_{[n]} \right) \quad (1)$$

where u, v are orthogonal coordinates of a point in a typical cross-section of the waveguide, z is the axis of the waveguide, $\Pi_{(n)}$ and $\Pi_{[n]}$ are the Hertz scalar functions of the TM and TE modes of the same waveguide filled with homogeneous isotropic achiral materials, h_1 and h_2 are the metrical coefficients, the V's and I's correspond to the voltages and currents in the coupled-mode waveguide. In fact, (1) is the expansion of the six field components separately into the summation of the eigenmode components of the same waveguide filled with homogeneous isotropic achiral material (There are six different sets of coefficients $V_{(n)}, V_{[n]}, \ldots$ in the expansion.) Of course, there are many possible ways to expand the six field components. Nevertheless, the above expansion gives a clear physical meaning and it makes the field calculations more convenient. The proposed method was successfully used and validated both theoretically and experimentally in the study of ferrite waveguides [17], [18]. Here we shall extend it to the study of bi-anisotropic waveguides, including chirowaveguides and Faraday chiral waveguides presented as special cases. A detailed discussion of expansion (1) is given in Appendix A.

From (1) and Maxwell's equations it is found that [17]

$$\frac{dV_{(m)}}{dz} = j\omega \int\int \left(B_u \frac{\partial \Pi_{(m)}}{h_2 \partial v} - B_v \frac{\partial \Pi_{(m)}}{h_1 \partial u} \right) ds + \chi_{(m)} V_{z,(m)}$$

$$\frac{dI_{(m)}}{dz} = -j\omega \int\int \left(D_u \frac{\partial \Pi_{(m)}}{h_1 \partial u} + D_v \frac{\partial \Pi_{(m)}}{h_2 \partial v} \right) ds$$

$$\frac{dV_{[m]}}{dz} = -j\omega \int\int \left(B_u \frac{\partial \Pi_{[m]}}{h_1 \partial u} + B_v \frac{\partial \Pi_{[m]}}{h_2 \partial v} \right) ds$$

$$\frac{dI_{[m]}}{dz} = j\omega \int\int \left(-D_u \frac{\partial \Pi_{[m]}}{h_2 \partial v} + D_v \frac{\partial \Pi_{[m]}}{h_1 \partial u} \right) ds + \chi_{[m]} I_{z,[m]} \quad (2)$$

and

$$V_{[m]} = -j\omega \int\int B_z \Pi_{[m]} ds$$

Manuscript received January 17, 1994; revised August 9, 1994.
The authors are with the Groupe de Recherches Avancées en Microondes et en Électronique Spatiale, (POLY-GRAMES), Département de Génie Électrique et de Génie Informatique, École Polytechnique de Montréal, Montréal, Québec, Canada, H3C 3A7.
IEEE Log Number 9408579.

0018–9480/95$04.00 © 1995 IEEE

$$I_{(m)} = -j\omega \int\int D_z \Pi_{(m)} ds \tag{3}$$

for $m = 0,1,2, \ldots$
where the surface integrals are performed across the cross-section of the waveguide.

The variation of the field components in the axial direction z is given by $\exp(-j\beta z)$. The constitutive relations in a most general bi-anisotropic medium for an assumed time dependence of the form $\exp(j\omega t)$ are given by [19]

$$\begin{aligned}\vec{B} &= \bar{\bar{\mu}}\vec{H} + \bar{\bar{\eta}}\vec{E}\\ \vec{D} &= \bar{\bar{\epsilon}}\vec{E} + \bar{\bar{\xi}}\vec{H}\end{aligned} \tag{4}$$

In principle, we may follow [17] to substitute (4) into (2), (3) and obtain all the transfer coefficients of the generalized coupled mode equations Z's, Y's, and T's which relate $I_{(n)}, I_{[n]}, V_{(n)}$ and $V_{[n]}$ to their derivatives $\frac{dI_{(m)}}{dz}$, $\frac{dI_{[m]}}{dz}$, $\frac{dV_{(m)}}{dz}$ and $\frac{dV_{[m]}}{dz}$.

To this end we have to substitute the expressions of B_z and D_z (which in general case are dependent on all the six field components E_u, E_v, E_z, H_u, H_v, and H_z) into (3); solve for $V_{z,(m)}$ and $I_{z,[m]}$ and eliminate them in (2). This is really a very complicated task, since the two infinite series of $V_{z,(m)}$ and $I_{z,[m]}$ are coupled to each other. Therefore, the following procedure is proposed to make our calculation practical. In fact, (3) are equivalent to

$$\begin{aligned}-j\omega B_z &= \sum_n \chi_{[n]}^2 V_{[n]} \Pi_{[n]}\\ -j\omega D_z &= \sum_n \chi_{(n)}^2 I_{(n)} \Pi_{(n)}\end{aligned} \tag{5}$$

and the last two equations of (1) may be transformed into

$$\begin{aligned}I_{z,[m]} &= \chi_{[m]} \int\int H_z \Pi_{[m]} ds\\ V_{z,(m)} &= \chi_{(m)} \int\int E_z \Pi_{(m)} ds.\end{aligned} \tag{6}$$

From (4) it is easy to obtain

$$\begin{aligned}\vec{B}_t &= \bar{\bar{\mu}}_t^* \vec{H}_t + \bar{\bar{\lambda}}_b \vec{B}_z + \bar{\bar{\eta}}_t^* \vec{E}_t + \bar{\bar{\chi}}_{tz} \vec{D}_z\\ \vec{D}_t &= \bar{\bar{\epsilon}}_t^* \vec{E}_t + \bar{\bar{\lambda}}_d \vec{D}_z + \bar{\bar{\xi}}_t^* \vec{H}_t + \bar{\bar{\sigma}}_{tz} \vec{B}_z\\ \vec{H}_z &= \bar{\bar{\lambda}}_h \vec{H}_t + \nu_z \vec{B}_z + \bar{\bar{\tau}}_{zt} \vec{E}_t + \psi_z \vec{D}_z\\ \vec{E}_z &= \bar{\bar{\lambda}}_e \vec{E}_t + \zeta_z \vec{D}_z + \bar{\bar{\varphi}}_{zt} \vec{H}_t + \varpi_z \vec{B}_z\end{aligned} \tag{7}$$

where $\vec{B}_t, \vec{D}_t, \vec{E}_t, \vec{H}_t$ are the field components in the cross-section plane of the waveguide, i.e. $\vec{B}_t = (\vec{B}_u, \vec{B}_v)^t$ etc. The expressions of the different coefficient matrices $\bar{\bar{\mu}}_t^*, \bar{\bar{\epsilon}}_t^*, \ldots$ are given in Appendix B. Eliminating $V_{z,(m)}$ and $I_{z,[m]}$ in (2) by using (6), we may express the right hand side of (2) by $\vec{H}_t, \vec{E}_t, \vec{B}_z$, and $\vec{D}_z$ after substituting (7) into them. These field components are then transformed into expressions of $\Pi_{[n]}$ and $\Pi_{(n)}$ through the first four equations of (1) and (5). By this way we may obtain all the transfer coefficients, containing only simple integrations and the propagation constact β will be easily solved by numerical calculation. In the following we shall take the chiral waveguide as an example. It is known that there are different constitutive relations of chiral materials [11]–[13] and a detailed discussion of them is given in [12], [13]. However, the various constitutive equations have been shown to be equivalent to each other for time-harmonic fields [12]. Here we take the following form of the constitutive relations to derive the expressions for chirowaveguide. This choice is arbitrary and it is easy to make the same derivations by using other types of constitutive relations. In this case, we have

$$\begin{aligned}\vec{B} &= \mu\vec{H} + j\xi_c\mu\vec{E}\\ \vec{D} &= \epsilon_c\vec{E} - j\xi_c\mu\vec{H}.\end{aligned} \tag{8}$$

By using the proposed procedure the coupled-mode equations for waveguide filled with nonmagnetic chiral materials (i.e. μ is constant across the waveguide) may be readily obtained as follows:

$$\begin{aligned}\frac{dV_{(m)}}{dz} &= -\sum_n \left(Z_{(m)(n)} I_{(n)} + T^V_{(m)(n)} V_{(n)} + T^V_{(m)[n]} V_{[n]}\right)\\ \frac{dI_{(m)}}{dz} &= -\sum_n \left(Y_{(m)(n)} V_{(n)} + Y_{(m)[n]} V_{[n]}\right.\\ &\qquad \left. + T^I_{(m)(n)} I_{(n)} + T^I_{(m)[n]} I_{[n]}\right)\\ \frac{dV_{[m]}}{dz} &= -Z_{[m][m]} I_{[m]} - \sum_n \left(T^V_{[m][n]} V_{[n]} + T^V_{[m](n)} V_{(n)}\right)\\ \frac{dI_{[m]}}{dz} &= -\sum_n \left(Y_{[m][n]} V_{[n]} + Y_{[m](n)} V_{(n)}\right.\\ &\qquad \left. + T^I_{[m][n]} I_{[n]} + T^I_{[m](n)} I_{(n)}\right)\end{aligned} \tag{9}$$

for $m = 0, 1, 2, \ldots$, and

$$\begin{aligned}Z_{(m)(n)} &= j\omega\mu + \int\int \left(\chi_{(m)}^2 \chi_{(n)}^2 \Pi_{(n)} \Pi_{(m)} / jw\epsilon\right) ds\\ Z_{[m][m]} &= j\omega\mu,\\ Y_{(m)(n)} &= j\omega \int\int \epsilon_c \left(\frac{\partial \Pi_{(n)}}{h_1 \partial u} \frac{\partial \Pi_{(m)}}{h_1 \partial u} + \frac{\partial \Pi_{(n)}}{h_2 \partial v} \frac{\partial \Pi_{(m)}}{h_2 \partial v}\right) ds,\\ Y_{(m)[n]} &= Y_{[n](m)}\\ &= j\omega \int\int \epsilon_c \left(\frac{\partial \Pi_{[n]}}{h_2 \partial v} \frac{\partial \Pi_{(m)}}{h_1 \partial u} - \frac{\partial \Pi_{[n]}}{h_1 \partial u} \frac{\partial \Pi_{(m)}}{h_2 \partial v}\right) ds\\ Y_{[m][n]} &= j\omega \int\int \epsilon_c \left(\frac{\partial \Pi_{[n]}}{h_1 \partial u} \frac{\partial \Pi_{[m]}}{h_1 \partial u} + \frac{\partial \Pi_{[n]}}{h_2 \partial v} \frac{\partial \Pi_{[m]}}{h_2 \partial v}\right) ds\\ &\quad + \frac{\chi_{[m]}^2 \chi_{[n]}^2}{jw\mu} \int\int (\epsilon_c \Pi_{[n]} \Pi_{[m]} / \epsilon) ds\\ T^V_{(m)(n)} &= T^I_{(m)(n)}\\ &= \omega\mu \int\int \xi_c \left(\frac{\partial \Pi_{(n)}}{h_1 \partial u} \frac{\partial \Pi_{(m)}}{h_2 \partial v} - \frac{\partial \Pi_{(n)}}{h_2 \partial v} \frac{\partial \Pi_{(m)}}{h_1 \partial u}\right) ds,\\ T^V_{[m][n]} &= T^I_{[m][n]}\\ &= -\omega\mu \int\int \xi_c \left(\frac{\partial \Pi_{[n]}}{h_2 \partial v} \frac{\partial \Pi_{[m]}}{h_1 \partial u} - \frac{\partial \Pi_{[n]}}{h_1 \partial u} \frac{\partial \Pi_{[m]}}{h_2 \partial v}\right) ds,\\ -T^V_{[m](n)} &= T^I_{(n)[m]}\\ &= \omega\mu \int\int \xi_c \left(\frac{\partial \Pi_{(n)}}{h_1 \partial u} \frac{\partial \Pi_{[m]}}{h_1 \partial u} + \frac{\partial \Pi_{(n)}}{h_2 \partial v} \frac{\partial \Pi_{[m]}}{h_2 \partial v}\right) ds,\end{aligned}$$

$$T^V_{(m)[n]} = -T^I_{[n](m)}$$
$$= \omega\mu \int\int \xi_c \left(\frac{\partial \Pi_{[n]}}{h_1 \partial u} \frac{\partial \Pi_{(m)}}{h_1 \partial u} + \frac{\partial \Pi_{[n]}}{h_2 \partial v} \frac{\partial \Pi_{(m)}}{h_2 \partial v} \right)$$
$$\times ds + \frac{\chi^2_{(m)} \chi^2_{[n]}}{w} \int\int \left(\xi_c \Pi_{[n]} \Pi_{(m)} / \epsilon \right) ds \quad (10)$$

where $\epsilon_c = \epsilon + \mu\xi_c^2$. Equations (9) and (10) are rigorous, without approximation. They are valid for cases where ξ_c and ϵ change across the cross-section of the waveguide. From (9) and (10) it is clear that the features of the coupled-mode equations for the chiral case are:

1) The transfer impedances Z and the transfer admittance Y of the chirowaveguide are the same as if the waveguide is filled by achiral materials with dielectric constant ϵ_c.
2) All eight voltage and current transfer coefficients T^V and T^I are not equal to zero and are proportional to ξ_c. This means that the mode coupling effect due to the chirality is present mostly through the T^V and T^I transfer coefficients.
3) The coupled-mode behavior of chirowaveguides is similar to the longitudinal magnetized ferrite waveguide [17] and the coupling coefficients are of the same form for these two cases. The main difference is that the coupling in chirowaveguides is present through coefficients T^V and T^I, and in ferrite waveguides through coefficients Z.

The coupled wave equations of ferrite waveguides were studied in [17] (for longitudinal magnetized case) and [18] (for transversely magnetized case), hence we shall not discuss them here.

Before performing the calculation of Faraday (ferrite) chiral waveguides, their constitutive relations should be examined first. Nevertheless, the study of the constitutive relations of this material is beyond the scope of this work. Moreover, at present the Faraday chiral material is a conceptual medium. Hence we restrict our study only to make some calculations on this type of waveguides for the purpose to demonstrate the effectiveness of the proposed method of calculation. In reference [16] the constitutive relations of Faraday (ferrite) chiral material are given by

$$\vec{B} = \bar{\bar{\mu}}\vec{H} + j\xi_c\bar{\bar{\mu}}\vec{E}$$
$$\vec{D} = \epsilon\vec{E} - j\xi_c\bar{\bar{\mu}}\vec{H} + \xi_c{}^2\, \bar{\bar{\mu}}\vec{E}. \quad (11)$$

In this paper we consider the longitudinally magnetized case, namely

$$\bar{\bar{\mu}} = \mu_1(\hat{u}\hat{u} + \hat{v}\hat{v}) - j\mu_a(\hat{u}\hat{v} - \hat{v}\hat{u}) + \mu_z\hat{z}\hat{z} \quad (12)$$

and $\mu_1 = \mu_z$. Applying the proposed method of solution, we may obtain the coupling coefficients which consist of the transfer impedances Z of the ferrite case (longitudinally magnetized), the transfer coefficients T of the chiral case, some terms of T which are proportional to the product $\xi_c\mu_a$ and some terms of the transfer admittances Y which are proportional to the product $\mu_a\xi_c^2$. For comparison, the following constitutive equations are also used:

$$\vec{B} = \bar{\bar{\mu}}\vec{H} + j\xi_c\mu_0\vec{E}$$
$$\vec{D} = \epsilon_c\vec{E} - j\xi_c\mu_0\vec{H} \quad (13)$$

and the expressions of permeability tensor remain the same as (12). The latter constitutive relations (13) also satisfy the conditions of energy conservation [23], namely $\bar{\bar{\xi}} = \bar{\bar{\eta}}^+$ in (4) where superscript + denotes transpose and complex conjugate. This set of constitutive relations is consistent with the Condon's equations [12] and tends toward the former if the higher order terms $\mu_a\xi_c$ and $\mu_a\xi_c^2$ in (11) may be neglected. For this set of constitutive conditions (13) the coupling coefficients are simply the combination of the ferrite case (transfer impedances Z) and the chiral case (transfer coefficients T). Some numerical calculations using the above two set of constitutive relations will be given in the next section.

III. Numerical Results of Chiral and Faraday Chiral Waveguides

The validation of the calculation for waveguides filled with ferrite samples of complicated configuration by using coupled wave equations was performed in [18] and here we shall validate the computation of chiral waveguide case. To this end we perform the study of a circular waveguide of radius R, homogeneously filled with a chiral medium of relative scalar permittivity $\epsilon_1 = \epsilon/\epsilon_0 = 1$, relative scalar permeability $\mu_1 = \mu/\mu_0 = 1$, and chiral admittance $\xi_c = 1$ mS and bounded by a perfect electric conductor. The calculation applies to the fundamental $HE_{1,1}$ and higher order $EH_{0,1}$ modes as shown in Fig. 1 together with the results obtained from reference [3]. It is clear that good agreement exits between the various results and they are also consistent with the results in [14] and the experimental findings in [15]. In reference [3] the circular dielectric chirowaveguide of radius R is modeled by a finite region extending to $\rho \approx 5\ R$ that is terminated with a PEC wall. This problem has been simulated by the proposed method to check its effectiveness. The material parameters are the same as in [3]: $\epsilon_1 = 1.1$, $\mu_1 = 1$, $\epsilon_2 = \mu_2 = 1$ and $\xi_c = 1$ mS. The calculated results are shown in Fig. 2. It is clear that our results are in good agreement with [3]. When the radius of the waveguide R becomes larger, the agreement becomes worse. This may be due to the field configurations of the waves along the circular open chirowaveguides are quite different from the fields of the empty reference waveguide and the present method appears less efficient. A further study of the propagation characteristics of rectangular waveguide, with dimensions $a \times b$, homogeneously filled with a chiral medium of the same parameters as the circular waveguide case in Fig. 1, is performed and the calculated dependence of the propagation constant of the fundamental mode β with respect to b/λ_0 for different values of ξ_c (where a/λ_0 is fixed to be 0.7 and λ_0 is the wavelength in free space) are shown in Fig. 3. It is observed that the phase shift of the chiral case with respect to the achiral one is strongly dependent on the height of the waveguide b and the chirality ξ_c, in spite of the fact that the phase constant and the field configuration

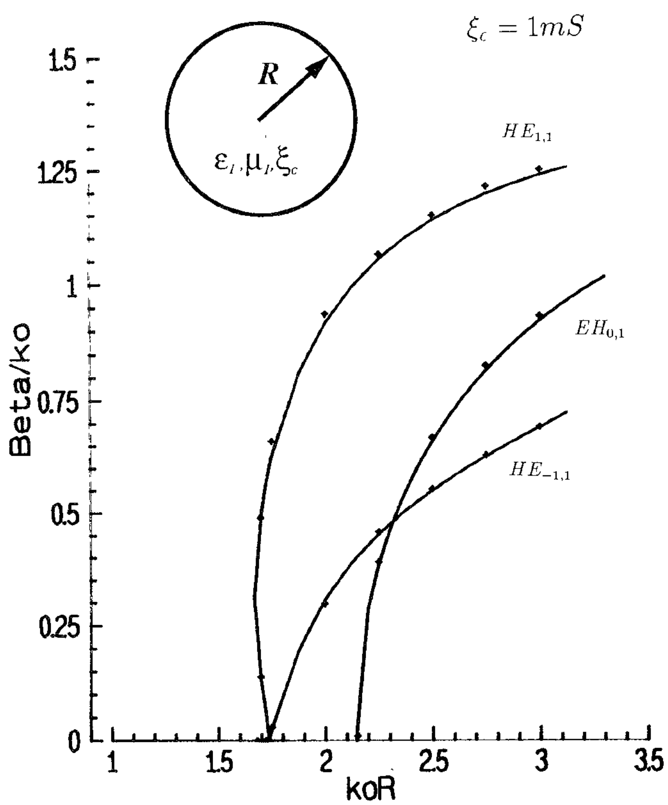


Fig. 1. Dispersion relations for the fundamental $HE_{1,1}$ and higher order $EH_{0,1}$ modes in the circular chirowaveguide. —: our results; + + ++: data from [3].

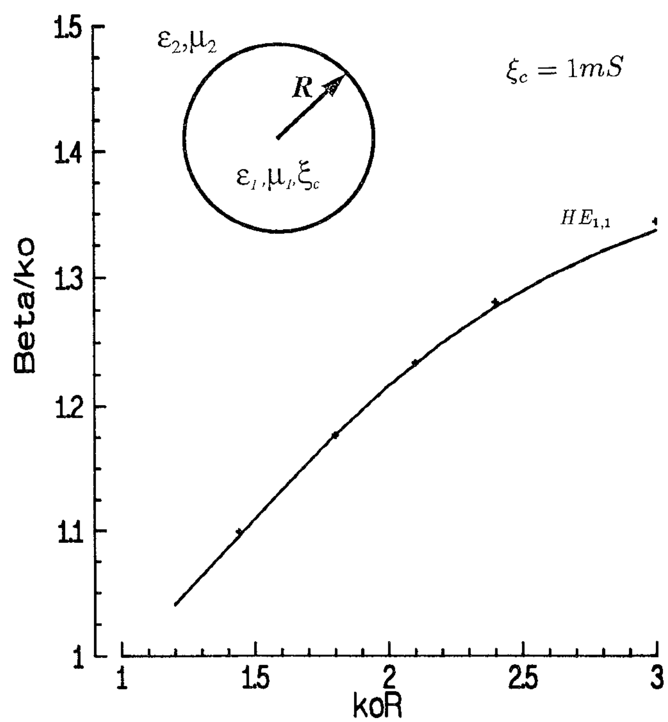


Fig. 2. Dispersion relations for the fundamental $HE_{1,1}$ mode in the circular dielectric chirowaveguide. —: our results; + + ++: data from [3].

of the fundamental mode are independent of b for the achiral case. This phenomenon is similar to the ferrite case—the phase shift of the longitudinal magnetized rectangular waveguide ferrite phase shifter remains insignificant when the height of the waveguide b is small and it increases very fast when the dimension b and the magnetizing field increase [21], [22].

Calculations for the Faraday (ferrite) chiral waveguides are performed by using the constitutive relations (11) and the calculated results for the circular waveguide case are plotted in Fig. 4. The nondiagonal element of the permeability tensor of the longitudinally magnetized Faraday (ferrite) chiral material μ_a/μ_0 equals ± 0.25 and the diagonal components of the permeability tensor are all equal to μ_1. The other material parameters are the same as in Fig. 1. It is observed that the change of the propagation constant of $HE_{1,1}$ mode for $\mu_a/\mu_0 = \pm 0.25$ is rather large and on the contrary, it appears small for $HE_{-1,1}$ mode. The criterion in classifying the modes in Faraday chiral waveguide is the same as in the chiral or ferrite waveguides, namely, HE-modes become H-modes when μ_a and ξ_c tend to zero and so on. It should be pointed out that the values of the propagation constants in this case remain unchanged if we change the signs of μ_a and ξ_c simultaneously and exchange the mode number, namely $HE_{-1,1} \longrightarrow HE_{1,1}$ and $HE_{1,1} \longrightarrow HE_{-1,1}$.

The curves of the same waveguide with the constitutive relations (13) are plotted in Fig. 5 for comparison where the values of ϵ, μ and ξ_c are the same as in Fig. 4. This comparison will show the effect of the higher order terms $\mu_a \xi_c$ and $\mu_a \xi_c^2$ in (11). It is seen that the general tendencies of the curves in

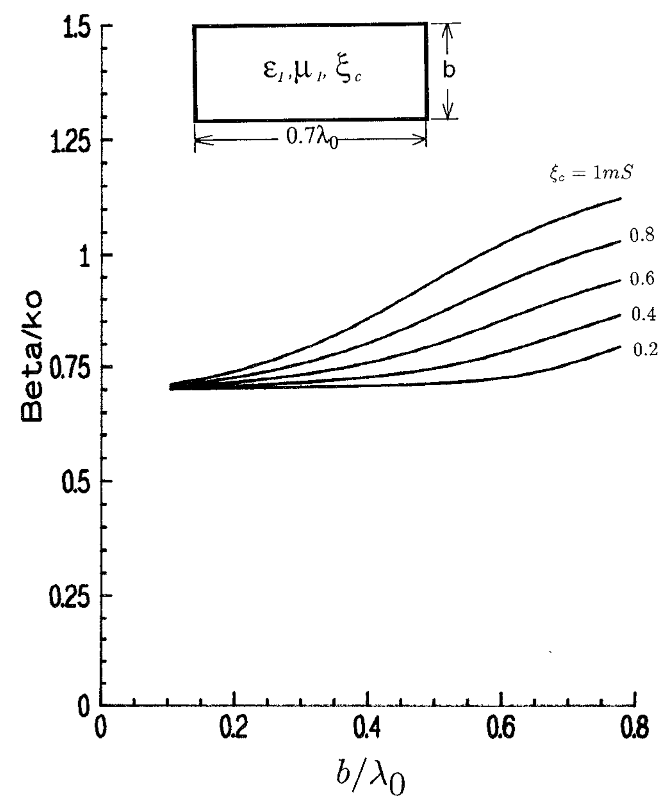


Fig. 3. Dependence of the propagation constant β/k_0 of the fundamental mode on the waveguide height b/λ_0 in the rectangular chirowaveguide with the chirality ξ_c as parameter.

Figs. 4 and 5 are the same but the differences of propagation constants of HE_{11} and HE_{-11} modes for $\mu_a/\mu_0 = \pm 0.25$ in Fig. 5 are much nearer to each other than in Fig. 4.

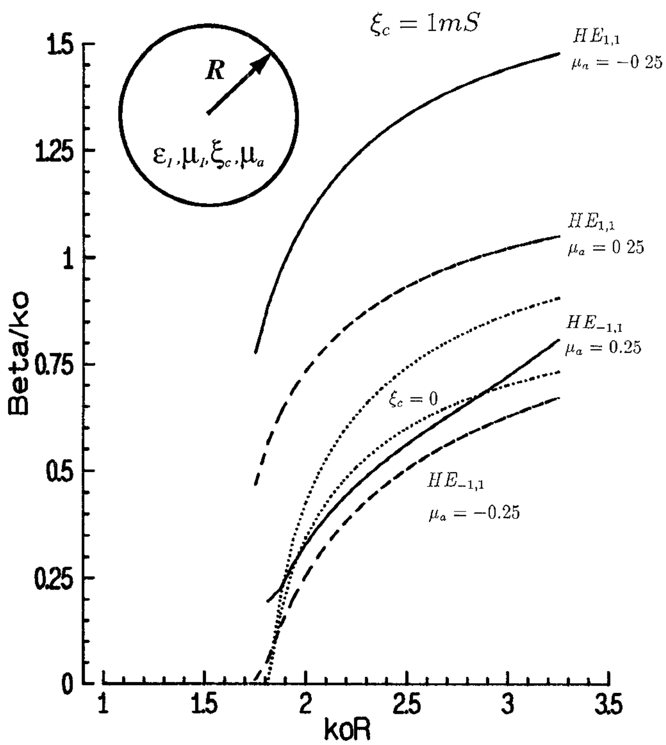


Fig. 4. Dispersion relations for the fundamental $HE_{1,1}$ modes in the circular Faraday (ferrite) chirowaveguide with $\mu_a/\mu_0 = \pm0.25$, $\xi_c = 1$ mS and constitutive relations (11). The dotted lines are with $\xi_c = 0$ for comparison.

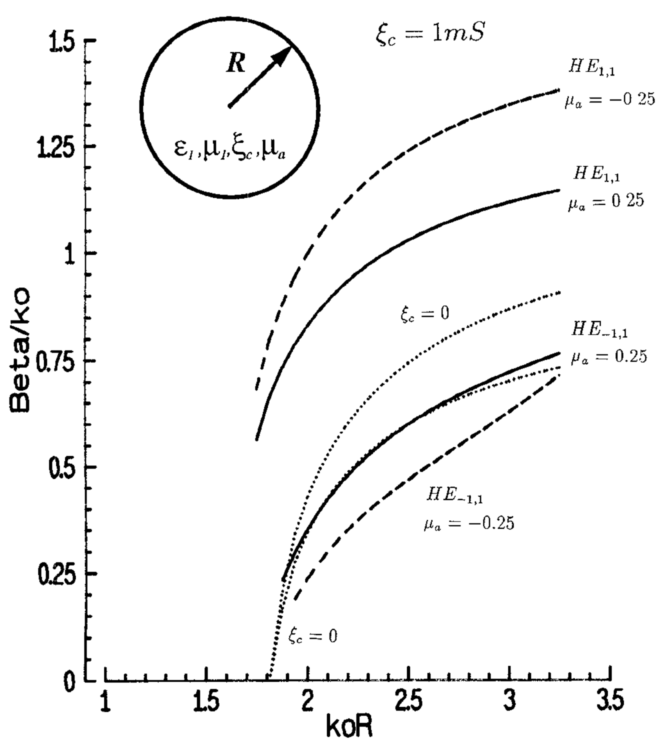


Fig. 5. Dispersion relations for the fundamental $HE_{1,1}$ modes in the circular Faraday (ferrite) chirowaveguide with $\mu_a/\mu_0 = \pm0.25$, $\xi_c = 1$ mS and constitutive relations (13) for comparison. The dotted lines are with $\xi_c = 0$.

To demonstrate the effectiveness of this method, the convergence characteristics of the calculation of Faraday (ferrite) chiral waveguide, chirowaveguide and dielectric chirowaveguide with circular cross-section are shown in Table I, II, and III, respectively. The total number of modes used in the calculation equals 4 $\times N$, where N is the largest number of variation of fields along the radius of the waveguide for the highest order mode considered in the calculation. Comparing the data of $N = 3$, 6, 12 and 24, we may find out that the convergence is good and the differences in convergence behavior for different cases are not significant. Nevertheless, it is clear that the convergence of the chirowaveguide case (Table II and III) is better then the Faraday (ferrite) chiral case (Table I) and differences of results of various values of N are smaller when the parameters ξ_c and μ_a become small (see Table I). This is quite reasonable. Generally speaking, for practical calculation it is sufficient to put $N = 5$ or 6, i.e. a total of 20 or 24 modes are considered.

TABLE I
CONVERGENCE CHARACTERISTICS OF THE CALCULATION OF THE PROPAGATION CONSTANT β/k_0 OF $HE_{1,1}$ MODE OF FARADAY (FERRITE) CHIRAL CIRCULAR WAVEGUIDES WITH $k_0R = 2.0$, $\epsilon = 1.0$

	N	2	3	4	5	6	12	24
ξ_c=1mS	μ_a=-0.5	1.2481	1.2549	1.2582	1.2602	1.2616	1.2650	1.2668
	μ_a=0.5	0.5416	0.5456	0.5479	0.5494	0.5504	0.5531	0.5546
ξ_c=2mS	μ_a=-0.5	2.1914	2.2077	2.2152	2.2196	2.2225	2.2297	2.2333
	μ_a=0.5	0.9186	0.9276	0.9325	0.9356	0.9377	0.9433	0.9462
ξ_c=2mS	μ_a=-0.75	2.4752	2.4930	2.5011	2.5059	2.5090	2.5167	2.5205
	μ_a=0.75	0.5479	0.5562	0.5610	0.5641	0.5663	0.5721	0.5751

TABLE II
CONVERGENCE CHARACTERISTICS OF THE CALCULATION OF THE PROPAGATION CONSTANT β/k_0 OF $HE_{1,1}$ MODE OF CHIRAL CIRCULAR WAVEGUIDES WITH $\epsilon = 1.0$ AND $\xi_c = 1$ ms

N	2	3	4	5	6	12	24
k_0R=3.6	1.3014	1.3025	1.3030	1.3032	1.3034	1.3039	1.3041
k_0R=4.0	1.3288	1.3298	1.3302	1.3304	1.3305	1.3309	1.3311

TABLE III
CONVERGENCE CHARACTERISTICS OF THE CALCULATION OF THE PROPAGATION CONSTANT β/k_0 OF $HE_{1,1}$ MODE OF CIRCULAR DIELECTRIC CHIROWAVEGUIDES WITH $\epsilon_1 = 1.1$ AND $\xi_c = 1$ ms

N	2	3	4	5	6	12	24
k_0R=3 6	1.1759	1.2636	1.3184	1.3466	1.3575	1.3562	1 3570
k_0R=4.0	1.1829	1.2750	1.3328	1.3624	1.3736	1.3706	1.3714

IV. CONCLUSION

The generalized coupled-mode equations for general bi-anisotropic wave guides are studied and derived. This method of calculation is very effective, flexible, simple and suitable to the most general bi-anisotropic material cases. This method of calculation is especially useful when we need to compute different cases, containing different kinds of materials (such as ferrite, chiral, Faraday chiral etc.) because this method needs only a modification in the relevant transfer coefficients in the coupled-wave equations in the computation. The case of waveguides, inhomogeneously filled with different bi-anisotropic materials can also be solved by using this method. To demonstrate its effectiveness the cases of Faraday (ferrite) chiral and chiral waveguides with both circular and rectangular cross-sections are studied and calculated curves are given. Interesting and practical important features of new cases are discovered and discussed.

Appendix A

In this appendix we shall show that (1) applies to any waveguide filled with a general bi-anisotropic medium and having PEC walls. The problem of electromagnetic field representations in spatially inhomogeneous bianisotropic media are investigated in [9]. It is shown that t he solution of Maxwell's equations can be reduced to the solution of a set of partial differential equations for four scalar potentials from which the fields can be derived. The tangential electromagnetic fields can be represented by this four scalar potentials as follows [9, (14)]:

$$\begin{aligned}\vec{E}^t(u,v) &= \nabla^t\Phi(u,v) + [\nabla^t \times \Theta(u,v)\vec{z}]\\ \vec{H}^t(u,v) &= \nabla^t\Upsilon(u,v) + [\nabla^t \times \Psi(u,v)\vec{z}].\end{aligned} \tag{A1}$$

It is clear that these two equations are identical to the first four equations in (1) after the substitutions $\Phi \longrightarrow \sum_n(V_{(n)}\Pi_{(n)}), \Theta \longrightarrow \sum_n(V_{[n]}\Pi_{[n]}), \Upsilon \longrightarrow \sum_n(I_{[n]}\Pi_{[n]})$ and $\Psi \longrightarrow \sum_n(V_{(n)}\Pi_{(n)})$. The longitudinal components E_z and H_z can be solved by substituting the tangential field components into Maxwell's equations. By simple common sense it is also clear that the most general representation of the expansions of the six field components may be expressed by six scalar potentials and no more is needed. (1) is exactly the case where six set of potentials $\sum_n(V_{(n)}\Pi_{(n)}), \sum_n(V_{[n]}\Pi_{[n]}), \sum_n(I_{(n)}\Pi_{(n)}), \sum_n(I_{[n]}\Pi_{[n]}), \sum_n(V_{z,(n)}\Pi_{(n)})$ and $\sum_n(I_{z,[n]}\Pi_{[n]})$ are present. The expansions in (1) is completely different from the so-called TE-to-z and TM-to-z decompositions which proved not possible in general chiral and biisotropic media by many references such as [8]. These decompositions mean that the electromagnetic fields may be expressed by using the sum of either single electric or single magnetic Hertz scalar potential function ($\Pi_{(n)}$ or $\Pi_{[n]}$). In contrary to this, we have six scalar potentials instead of one in (1). Moreover, here we use the Hertz scalar functions only because these functions are very familiar to us, for example, in rectangular or circular waveguides filled with homogeneous achiral isotropic materials these potentials are related to the well-known E- and H-modes in these waveguides. In fact, we may take other forms of expansions instead of using Hertz potentials, only if they are convenient for us. The reference waveguides whose Hertz potentials are used in (1) may also be different. Nevertheless, for convenience we usually choose the waveguides with the same configurations as the bi-anisotropic one but filled with air. This choice fulfils the boundary conditions required and makes the Hertz potentials very simple to improve the efficiency of the numerical calculation.

As for the completeness of the representations in (1), it may be checked by the features of convergence and accuracy tests in the numerical calculation. The numerical examples of this paper show that both of these two features are very satisfactory and hence the completeness of the representations in (1) should be of no problem. Generally speaking, the boundary conditions and the differential equations ruling the bi-anisotropic problem and the isotropic problem are the same, the difference lies only in the constitutive relations, namely, the relationship between B, D and E, H. Therefore, it is reasonable to construct the solution of the bi-anisotropic case from the sum of the solutions of the isotropic case. The effectiveness of the solution of different waveguides, inhomogeneously filled with dielectrics and gyrotropic ferrites in the literature also proves that the expansions in (1) are adequate.

Appendix B

After partition the tensors $\bar{\bar{\epsilon}}, \bar{\bar{\mu}}, \bar{\bar{\eta}}$ and $\bar{\bar{\xi}}$ in (4) we have the following expressions:

$$\begin{aligned}\vec{B}_t &= \bar{\bar{\mu}}_t\vec{H}_t + \bar{\bar{\mu}}_z\vec{H}_z + \bar{\bar{\eta}}_t\vec{E}_t + \bar{\bar{\eta}}_z\vec{E}_z\\ \vec{D}_t &= \bar{\bar{\epsilon}}_t\vec{E}_t + \bar{\bar{\epsilon}}_z\vec{E}_z + \bar{\bar{\xi}}_t\vec{H}_t + \bar{\bar{\xi}}_z\vec{H}_z\\ \vec{B}_z &= \bar{\bar{\mu}}^z\vec{H}_t + \mu_{zz}\vec{H}_z + \bar{\bar{\eta}}^z\vec{E}_t + \eta_{zz}\vec{E}_z\\ \vec{D}_z &= \bar{\bar{\epsilon}}^z\vec{E}_t + \epsilon_{zz}\vec{E}_z + \bar{\bar{\xi}}^z\vec{H}_t + \xi_{zz}\vec{H}_z\end{aligned} \tag{B1}$$

with

$$\|\bar{\bar{\mu}}\| = \begin{Vmatrix}\bar{\bar{\mu}}_t & \bar{\bar{\mu}}_z\\ \bar{\bar{\mu}}^z & \mu_{zz}\end{Vmatrix} \tag{B2}$$

and so forth. Solving for $\vec{E}_z$ and $\vec{H}_z$ from the last two equations of (B-1) and substituting them into the first two, we may obtain the following expressions for (7):

$$\begin{aligned}\bar{\bar{\mu}}_t^* &= \bar{\bar{\mu}}_t + [\bar{\bar{\mu}}_z(\eta_{zz}\bar{\bar{\xi}}^z - \epsilon_{zz}\bar{\bar{\mu}}^z) + \bar{\bar{\eta}}_z(\xi_{zz}\bar{\bar{\mu}}^z - \mu_{zz}\bar{\bar{\xi}}^z)]/\alpha\\ \bar{\bar{\eta}}_t^* &= \bar{\bar{\eta}}_t + [\bar{\bar{\mu}}_z(\eta_{zz}\bar{\bar{\epsilon}}^z - \epsilon_{zz}\bar{\bar{\eta}}^z) + \bar{\bar{\eta}}_z(\xi_{zz}\bar{\bar{\eta}}^z - \mu_{zz}\bar{\bar{\epsilon}}^z)]/\alpha\\ \bar{\bar{\epsilon}}_t^* &= \bar{\bar{\epsilon}}_t + [\bar{\bar{\epsilon}}_z(\xi_{zz}\bar{\bar{\eta}}^z - \mu_{zz}\bar{\bar{\epsilon}}^z) + \bar{\bar{\xi}}_z(\eta_{zz}\bar{\bar{\epsilon}}^z - \epsilon_{zz}\bar{\bar{\eta}}^z)]/\alpha\\ \bar{\bar{\xi}}_t^* &= \bar{\bar{\xi}}_t + [\bar{\bar{\xi}}_z(\eta_{zz}\bar{\bar{\xi}}^z - \epsilon_{zz}\bar{\bar{\mu}}^z) + \bar{\bar{\epsilon}}_z(\xi_{zz}\bar{\bar{\mu}}^z - \mu_{zz}\bar{\bar{\xi}}^z)]/\alpha\end{aligned} \tag{B3}$$

and

$$\begin{aligned}\bar{\bar{\lambda}}_b &= (\bar{\bar{\mu}}_z\epsilon_{zz} - \bar{\bar{\eta}}_z\xi_{zz})/\alpha\\ \bar{\bar{\chi}}_{tz} &= (-\bar{\bar{\mu}}_z\eta_{zz} + \bar{\bar{\eta}}_z\mu_{zz})/\alpha\\ \bar{\bar{\lambda}}_d &= (\bar{\bar{\epsilon}}_z\mu_{zz} - \bar{\bar{\xi}}_z\eta_{zz})/\alpha\\ \bar{\bar{\sigma}}_{tz} &= (-\bar{\bar{\epsilon}}_z\xi_{zz} + \bar{\bar{\xi}}_z\epsilon_{zz})/\alpha\\ \bar{\bar{\lambda}}_h &= (\eta_{zz}\bar{\bar{\xi}}^z - \epsilon_{zz}\bar{\bar{\mu}}^z)/\alpha\\ \bar{\bar{\tau}}_{zt} &= (\eta_{zz}\bar{\bar{\epsilon}}^z - \epsilon_{zz}\bar{\bar{\eta}}^z)/\alpha\\ \bar{\bar{\lambda}}_e &= (\xi_{zz}\bar{\bar{\eta}}^z - \mu_{zz}\bar{\bar{\epsilon}}^z)/\alpha\\ \bar{\bar{\varphi}}_{zt} &= (\xi_{zz}\bar{\bar{\mu}}^z - \mu_{zz}\bar{\bar{\xi}}^z)/\alpha\end{aligned}$$

$$\begin{aligned}\nu_z &= \epsilon_{zz}/\alpha \\ \psi_z &= -\eta_{zz}/\alpha \\ \zeta_z &= \mu_{zz}/\alpha \\ \varpi_z &= -\xi_{zz}/\alpha\end{aligned} \tag{B4}$$

where $\alpha = \epsilon_{zz}\mu_{zz} - \eta_{zz}\xi_{zz}$ and the only condition for this transform is $\alpha \neq 0$.

REFERENCES

[1] N. Engheta and P. Pelet, "Modes in chirowaveguides," *Opt. Lett.*, vol. 14, pp. 593–595, June 1989.

[2] ——, "Mode orthogonality in chirowaveguides," *IEEE Trans. Microwave Theory Tech.* vol. MTT-38, pp. 1631–1634, Nov. 1990.

[3] J. M. Svedin, "Propagation analysis of chirowaguides using the finite-element method," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-38, pp. 1488–1496, Oct. 1990.

[4] P. Pelet and N. Engheta, "The Theory of chirowaveguide," *IEEE Trans. Antenn. Propagat.*, vol. AP-38, pp. 91–98, Jan. 1990.

[5] T. C. K. Rao, "Attenuation characteristics of a circular chirowaveguide," *Electron. Lett.*, vol. 26, pp. 1767–1769, Oct. 1990.

[6] C. Eftimiu and L. W. Pearson, "Guided electromagnetic waves in chiral media," *Radio Sci.*, vol. 24, no 3, pp. 351–359, May–June 1989.

[7] P. Pelet and N. Engheta, "Coupled-mode theory for chirowaveguides," *J. Appl. Phys.*, vol. 67, pp. 2742–2745, Mar. 1990.

[8] V. K. Varadan, A. Lakhlesh, and V. V. Varadan, "Propagation in a parallel-plate waveguide wholly filled with a chiral medium," *J. Wave-Material Interation*, vol. 3, pp. 267–272, July 1988; "Errata," *Ibid*, vol. 3, p. 351, 1988.

[9] W. Weiglhofer, "Scalarisation of Maxwell's equations in general inhomogeneous bianisotropic media," *IEE Proc.*, vol. 134, pt. H, pp. 357–360, 1987.

[10] E. J. Post, *Formal Structure of Electromagnetics*. Amsterdam, The Netherlands: North-Holland, 1962.

[11] D. L. Jaggard, A. R. Mickelson, and C. H. Papas, "On electromagnetic waves in chiral media," *Appl. Phys.*, vol. 18, pp. 211–216, 1979.

[12] A. Lakhtakia, V. V. Varadan, and V. K. Varadan, "Time-harmonic electromagnetic fields in chiral media," in *Lecture Notes in Physics No. 335*. Berlin: Springer-Verlag, 1989.

[13] A. H. Sihvola and I. V. Lindell, "Bi-isotropic constitutive relations," *Microwave Opt. Tech. Lett.*, vol. 4, no. 8, pp. 295–297, 1991.

[14] R. D. Hollinger, V. V. Varadan, and V. K. Varadan, "Eigenmodes in a circular waveguide containing an isotropic chiral material," *Radio Sci.*, vol. 26, no. 2, pp. 1335–1344, 1991.

[15] R. D. Hollinger, V. V. Varadan, D. K. Ghodgaonkar and V. K. Varadan, "Experimental characterization of isotropic chiral composites in circular waveguides," *Radio Sci.*, vol. 27, no. 2, pp. 161–168, 1992.

[16] N. Engheta, D. L. Jaggard, and M. W. Kowarz, "Electromagnetic waves in Faraday chiral media," *IEEE Trans. Antenn. Propagat.*, vol. AP-40, pp. 367–374, Apr. 1992.

[17] S. A. Schelkunoff, "Generalized Telegraphist's equations for waveguides," *Bell Syst. Tech. J.*, pp. 784–801, July 1952.

[18] Y. Xu and G. Zhang, "A rigorous method for computation of ferrite toroidal phase shifters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-36, pp. 929–933, June 1988.

[19] J. A. Kong, *Electromagnetic Wave Theory*. New York: Wiley, 1986.

[20] B. Lax and K. J. Button, *Microwave Ferrites and Ferrimagnetics*. New York: McGraw-Hill, 1962, section 12–7.

[21] A. Clavin, " Reciprocal ferrite phase shifters in rectangular waveguides," *IRE Trans. Microwave Theory Tech.*, vol. MTT-6, p. 334, 1958.

[22] J. A. Weiss, "A phenomenological theory of the Reggia-Spencer phase shifter," in *Proc. IRE*, vol. 47, p. 1130, 1959.

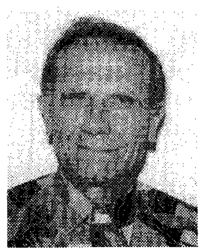
[23] J. A. Kong, "Theorems of bianisotropic media," *Proc. IEEE*, vol. 60, pp. 1036–1046, 1972.

[24] H. Cory and T. Tamir, "Coupling processes in circular open chirowaveguides," *IEE Proc.*, vol. 139, pt. H, pp. 165–170, 1992.

Yansheng Xu (M'93) graduated from Tsing Hua University, Beijing, China in 1952 and he received the degree of Candidate of Technical Science from the Institute of Radio Physics and Electronics, Academy of Science, Moscow, Russia, in 1961.

In 1961 he joined the Beijing Institute of Radio Measurement. His main research interests include microwave measurement techniques, numerical methods of analysis and computer-aided design of MMIC's and MHMIC's, electromagnetic fields, microwave ferrite devices and chiral materials.

Mr. Xu is a Fellow of the Chinese Institute of Electronics (CIE) and he has published over 80 technical papers in his field of interest.



Renato G. Bosisio (M'79-SM'89) received the B.Sc. degree from McGill University, Montréal, Canada, and the M.S.E.E. degree from the University of Florida, Gainesville..

He has been engaged in microwave research and development work with various firms: Marconi and Varian in Canada, Sperry in the US, and English Electric in England. He is presently head of Poly-GRAMES, University Microwave Research Group at Ecole Polytechnique de Montréal, Montréal, Canada, where he teaches microwave theory and techniques. He is actively engaged in six-port technology, dielectric measurements, and computer-aided testing and design of active and passive microwave devices in M(H)MIC.